Tutorial 6

Attempt all problems beforehand to engage in class discussion. No detailed written answers will be made available on Blackboard. We will go through them in class.

Tutorial exercise problems

N.B. The following questions cover the examinable material for the mid-semester test: questions 1, 2(a), 3, 4(a), 5(a) & 5(b). No written answer will be available before we go through them in class.

1. Consider an asset with annual volatility (σ) of 40% with the market beta of 1.2. Suppose that the annual volatility of the market is 25%. What percentage of the total volatility of the asset is attributable to non-systemic risk?

2. Consider the following utility function

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \rightarrow u'(c) = c^{-\gamma}$$

(a) Show that the relative risk aversion coefficient is given by γ. How do you interpret this coefficient?

The elasticity of intertemporal substitution in consumption (i.e. the willingness to shift consumption across time) is defined as

$$\varepsilon = -\frac{\partial(c_1/c_2)}{\partial(p_1/p_2)} \cdot \frac{(p_1/p_2)}{(c_1/c_2)} \text{ or } -\frac{\partial \ln(c_1/c_2)}{\partial \ln(p_1/p_2)}$$

Note that if you let the price of period 1 consumption be one unit the price of period 2 consumption is 1/(1+*r*), where *r* is the real interest rate in units of consumption.

(b) The intertemporal efficiency in consumption is given by the equation (also known as the consumption Euler equation)

$$u'(c_1) = \beta(1 + r)\, u'(c_2)$$

Use the above utility function to show that the elasticity of intertemporal substitution (ε) is the reciprocal of the relative risk aversion parameter (γ). What does this mean?

3. Assume that the utility function is $u(c) = \ln c$. Note that the marginal utility, $MU(c)$, is given by $u'(c) = 1/c$. Suppose that $c_1 = 1$ but the outcome of c_2 is contingent on the state (s) of the economy, as follows.

Bad (s_1)	$c_2 = 0.5$	Probability = 0.5
Good (s_2)	$c_2 = 1.5$	Probability = 0.5

(a) Calculate the implied rate of return for the one-period discount bond, with a guaranteed payoff of 1 consumption unit in period 2. Assume that the time preference factor β = 0.95.

(b) Now, consider a stock with the payoff structure that gives you the above state contingent consumption. That is, this asset is *perfectly correlated* with the economy and provides total consumption stream in period 2 as shown above. Determine the price of this stock and the expected return.

4. Consider a two-period economy with given incomes y_1 and y_2. In equilibrium, $c_1 = y_1$ and $c_2 = y_2$. Suppose that life time expected utility is given by

$$U(c_1, c_2) = \frac{1}{1-\gamma} c_1^{1-\gamma} + \beta \frac{1}{1-\gamma} c_2^{1-\gamma}, \quad \text{where } \gamma > 0.$$

(a) Write the consumption Euler equation linking between c_1 and c_2.

(b) The consumption-CAPM (C-CAPM) defines the stochastic discount factor or pricing kernel as

$$m_{t+1} = \beta u'(c_{t+1}) / u'(c_t)$$

from the asset pricing version of the Euler equation, $1 = E_t[(1+r)m_{t+1}]$.

What is the economic interpretation of m? Find m using the above utility function.

(c) Consider the price of q_1 of a riskless asset such as a non-state contingent bond, which pays 1 in any state in period 2. Suppose that c_1 and c_2 can take on any two values, 1 and 2, each with probability 0.5 and that c_1 and c_2 are independently distributed. Suppose that $\beta = 1$ and also assume that $\gamma = 2$. What is the variance of the interest rate on bond?

(d) Now suppose that $\gamma = 4$. Again find the variance of the interest rate. Is it larger than the variance of the gross consumption rate (c_2/c_1)?

5. One challenge for the C-CAPM is to explain that equities (stocks) tend to have higher returns than bonds. This question links each type of return to aggregate, real activity. Consider a two-period economy with a representative consumer (investor) whose consumption Euler (i.e. intertemporal efficiency) equation is:

$$\frac{1}{c_1} = E_1 \beta (1 + r_i) \frac{1}{c_2},$$

where r_i is the return on any asset. Let $\beta = 0.9$. Suppose that $c_1 = 1$ and let $c_2 = 1.1$ with probability 0.5 and 0.9 with probability 0.5.

(a) Find the price q and the return r on a riskless bond which pays 1 in each state in period 2.

(b) Now find the price $\bar{q}$ and the expected return $\bar{r}$ on an equity which pays c_2 (*i.e.* here it pays 1.1 when $c_2 = 1.1$ and it pays 0.9 when $c_2 = 0.9$), so that it is a claim to the aggregate consumption stream.

(c) Two empirically unrealistic features of our results so far are the low equity premium $\bar{r} - r$ and the high riskfree rate r. Show the effect on both of a larger variance of c_2. (Hint: Do not change the probabilities, but raise the large value and lower the small value, leaving the mean unchanged.)